Attitude Control of Satellites Using the Solar Radiation Pressure

V. J. MODI* and K. KUMAR†
The University of British Columbia, Vancouver, Canada

Introduction

THE gravity gradient stabilization technique, although conceptually simple, has a severe limitation in terms of pointing accuracy which further deteriorates under the perturbing influence of environmental disturbances. On the other hand, active attitude control methods assure accurate station-keeping but with penalty in terms of weight, satellite life-time and reliability. Keeping this in view, attempts have been made to develop attitude control procedures utilizing the environmental forces. In particular, the possibility of using the solar radiation pressure to advantage has been studied at length.[1-3] The authors[4] have investigated the feasibility of this concept in achieving the planar as well as the spatial librational damping and orientation control of satellites in circular ecliptic orbits.

This Note studies a further generalization of the controller performance for the satellite orbit in an arbitrary plane thus extending its usefulness to more realistic situations. The precise pointing accuracy, possibility of controlled change of satellite attitude in orbit together with low power consumption make the system quite attractive for the next generation of communications satellites and future space stations.

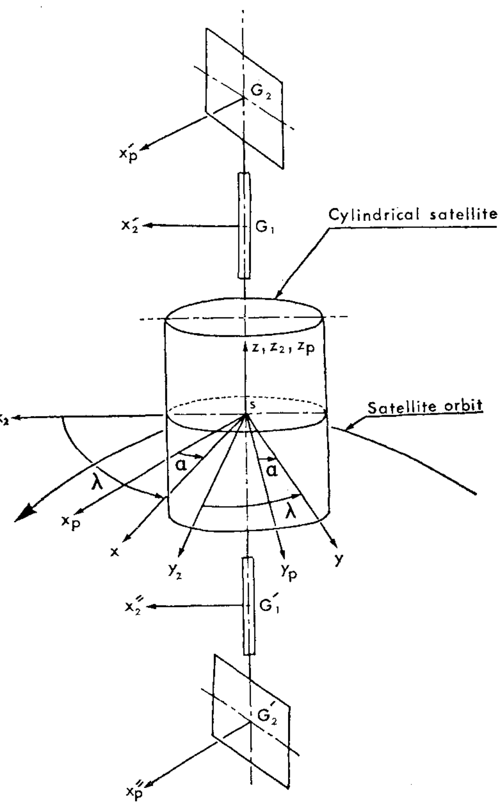

Fig. 1 Geometry of the proposed solar controller for general three-dimensional control.

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* Professor. Member AIAA.

† Graduate Research Fellow. Student Member AIAA.

Proposed Controller Configuration and Equations of Motion

The suggested controller model (Fig. 1) essentially consists of two sets of control plates, $G_1 - G_1'$ and $G_2 - G_2'$, made of a highly reflective, light but rigid material such as aluminized Mylar. The angular position of the sets, with relative inclination of 45°, remains unaffected during axial rotation of the satellite. The plates G_1, G_1' are constrained to move along the z_2 and y_2 axes whereas the plates G_2, G_2' are free to move only along the z_2 axis. Using the Lagrangian formulation with the generalized radiation forces, the governing equations for pitch (ψ), roll (β) and yaw (λ) librations can be written as[4]

$$\psi'' - 2(1+\psi')\beta'\tan\beta + 3K_i\sin\psi\cos\psi - (1-K_i)\frac{\sin\beta}{\cos^2\beta}\cdot\frac{d}{d\theta}\{\lambda' - (1+\psi')\sin\beta\} - (1-K_i)\frac{\beta'}{\cos\beta}\{\lambda' - (1+\psi')\sin\beta\} =$$
$$\Big[\frac{c_\psi\cos\beta + c_\lambda\sin\beta}{\cos^2\beta}\cdot\{\sin(\theta+\psi-\phi) + \alpha_1\cos(\theta+\psi)\}\cdot|\{\sin(\theta+\psi-\phi)+\alpha_1\cos(\theta+\psi)\}| +$$
$$\frac{c_\beta}{8^{1/2}\cos\beta}\cdot\{(1+2p)[\sin(\theta+\psi-\phi) + \alpha_1\cos(\theta+\psi)] - (1-2p)[\sin\beta\cos(\theta+\psi-\phi) - \alpha_1\sin\beta\sin(\theta+\psi) - \alpha_2\cos\beta]\}\cdot|[\sin(\theta+\psi-\phi) - \sin\beta\cos(\theta+\psi-\phi) + \alpha_1\{\cos(\theta+\psi)+\sin\beta\sin(\theta+\psi)\} + \alpha_2\cos\beta]|$$

$$\beta'' + \{(1+\psi')^2 + 3K_i\cos^2\psi\}\sin\beta\cos\beta + (1-K_i)(1+\psi')\cos\beta\{\lambda' - (1+\psi')\sin\beta\} =$$
$$2p[c_\psi\{\sin\beta\cos(\theta+\psi-\phi) - \alpha_1\sin\beta\sin(\theta+\psi) - \alpha_2\cos\beta\} - c_\lambda\{\cos\beta\cos(\theta+\psi-\phi) - \alpha_1\cos\beta\sin(\theta+\psi) + \alpha_2\sin\beta\}]\cdot|\{\sin(\theta+\psi-\phi) + \alpha_1\cos(\theta+\psi)\}| +$$
$$\frac{c_\beta}{(8)^{1/2}}[-(1-2p)\{\sin(\theta+\psi-\phi) + \alpha_1\cos(\theta+\psi)\} + (1+2p)\{\sin\beta\cos(\theta+\psi-\phi) - \alpha_1\sin\beta\sin(\theta+\psi) - \alpha_2\cos\beta\}].|[\sin(\theta+\psi-\phi) - \sin\beta\cos(\theta+\psi-\phi) + \alpha_1\{\cos(\theta+\psi) + \sin\beta\sin(\theta+\psi)\} + \alpha_2\cos\beta]$$

$$\lambda'' - \psi''\sin\beta - (1+\psi')\beta'\cos\beta = -\frac{c_\lambda}{(1-K_i)}\cdot[\sin(\theta+\psi-\phi) + \alpha_1\cos(\theta+\psi)]|[\sin(\theta+\psi-\phi) + \alpha_1\cos(\theta+\psi)]|$$

where θ = position of the satellite in an orbit as measured from the perigee; K_i = inertia parameter, $(I - I_{zz})/I$; ϕ = solar aspect angle, position of the sun with respect to the vernal equinox; i = inclination of the orbital plane with respect to the ecliptic; $\alpha_1 = (1-\cos i)\sin\phi$; $\alpha_2 = -\sin i\sin\phi$ and primes denote differentiation with respect to θ. The rest of the symbols have the same meaning as in Ref. 4.

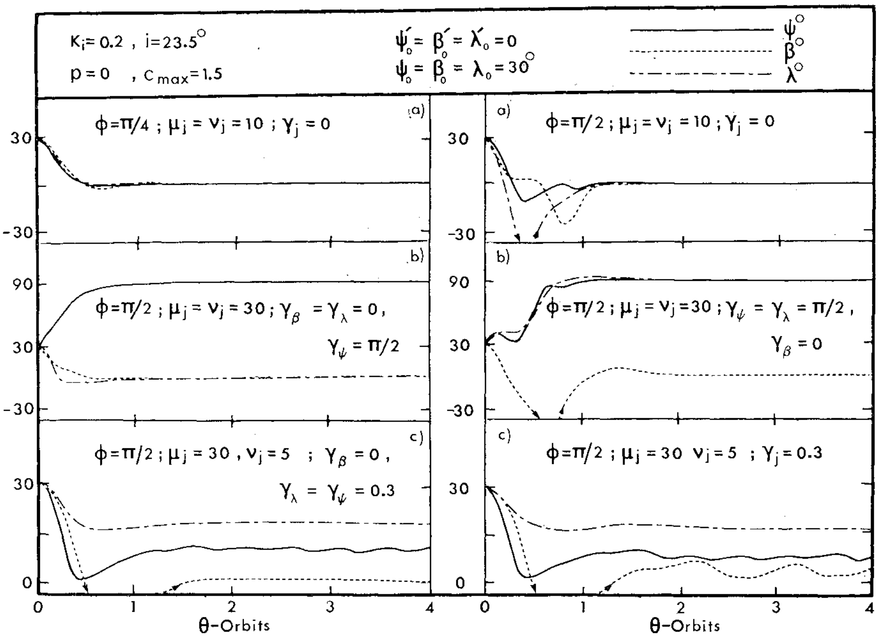

Fig. 2 Typical examples of satellite response showing the general three-dimensional attitude control using the proposed solar controller.

The generalized forces in the equations of motion are controlled according to the relations

$$c_j = \begin{cases} (-1)^n \quad [\mu_j j' + \nu_j (j - \gamma_j)]; \\ \qquad\qquad 2k\pi < (\theta + \psi - \phi) \leqslant (2k+1)\pi \\ (-1)^{n+1}[\mu_j j' + \nu_j (j - \gamma_j)]; \\ \qquad\qquad (2k+1)\pi < (\theta + \psi - \phi) \leqslant (2k+2)\pi \end{cases}$$

with $|c_j| < c_{\max}$, k denoting an integer, and $j = \psi, \beta, \lambda$. The integer n is odd for ψ and even for β and λ. μ_j and ν_j are the gains in the controller characteristic relations and γ_j refer to the position control angles.

Results and Discussions

The governing nonlinear, nonautonomous and coupled differential equations with periodic coefficients represent an extension of the simpler equations, obtained by Modi and Brereton,[5] to account for the solar radiation environmental effect. However, there are several important, far-reaching differences: 1) The spin degree of freedom (λ) does not represent a cyclic co-ordinate, hence the associated momentum is no longer a constant of the motion. 2) The nonautonomous character of the system leads to the Hamiltonian which is no more a constant of the system. The concept of integral manifold, which was exploited so fruitfully by Modi and Brereton[5] in analyzing the undamped, autonomous system, is of little use here. 3) Even for an undamped, nonautonomous system, the invariant hypersurface would present formidable problems of representation and interpretation.

In such a situation, numerical techniques were used to advantage to analyze the damping and attitude control characteristics of the system. The integration was performed using the Adams-Bashforth predictor corrector technique in conjunction with the Runge-Kutta starter taking the step-size of 3°.

Figure 2 shows a few typical situations in which the proposed solar controller attempts to provide the satellite desired

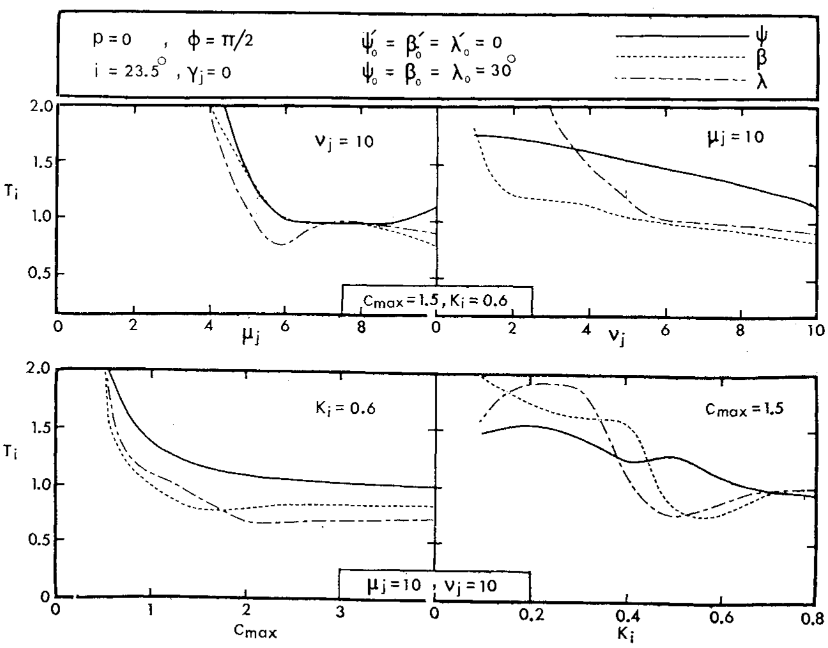

Fig. 3 System plots showing the damping characteristics of satellite for attitude control along the local vertical.

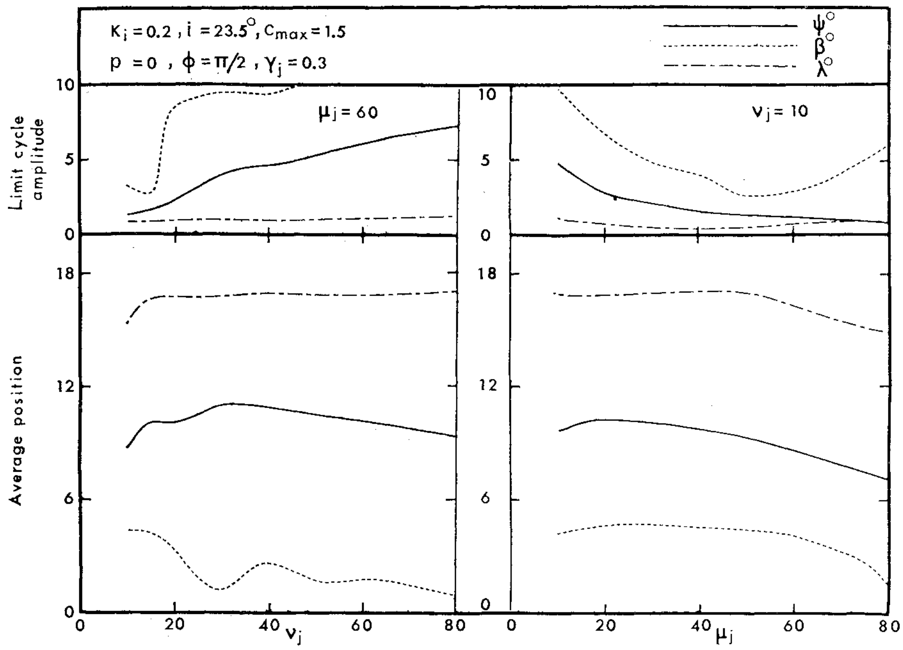

Fig. 4 System plots showing the attitude control characteristics for achieving desired spatial satellite orientation.

orientations. It becomes evident that the controller is successful in positioning the satellite precisely along the local vertical (Fig. 2a), as well as the local horizontal (Fig. 2b) representing the unstable equilibrium configuration for the gravity oriented systems. Furthermore, through a suitable choice of the position control parameters γ_j, intermediate orientations can also be attained (Fig. 2c). Numerous response plots were obtained to assess the influence of various system parameters on damping and attitude control characteristics. The resulting information is summarized in Figs. 3 and 4.

Influence of the controller gains (μ_j, ν_j), the maximum available control torque ($\propto c_{max}$), and satellite inertia distribution parameter (K_i) on the time-index T_i (time to damp the librational motion to within $\pm 0.5°$ expressed as a fraction of the orbital period) is shown in Fig. 3. In general, increase in controller gains and c_{max} affect the time-index favorably to a point, beyond which the dependence becomes somewhat weak. Longer satellites (large K_i) withstand the disturbance better and quickly regain the equilibrium position compared to short and stubby designs. The semipassive controller is able to damp the librational motion within a fraction of an orbit, even when the system is subjected to such a large disturbance.

Similar results were also obtained for the alignment of the satellite along the local horizontal. The time-index variation with c_{max} essentially followed the similar trend. However, as can be expected, higher gains were required to capture the satellite in this configuration. Here, as the control moment has to overcome the gravity gradient torque, the shorter satellites showed better performance.

For intermediate orientations, opposing influence of the gravity gradient and the control torques leads to a limit cycle type of response. The periodic character of the motion was established through the phase plane representation of the response over a large number of orbits. Here, it would be useful to record steady-state amplitudes and average equilibrium positions (Fig. 4). Importance of the position control parameters γ_j becomes apparent. Now the satellite is able to change its orientation in orbit thus enabling it to undertake diverse missions. In fact, a suitable selection of controller gains and c_{max} can lead to any desired spatial orientation. In general, the penalty for this versatility would be in terms of higher limit cycle amplitude. However, depending upon the mission, a judicious choice of parameters would normally limit it to an acceptable value.

Success of the proposed controller presents an exciting possibility for station-keeping of communications satellites. Size of the plates required is rater modest and so is the power consumption of the servo-system. A preliminary estimate for INTELSAT IV series of satellites showed that the plate size of 7 ft^2 with permissible movements of 1 ft ($c_{max} = 1.5$) would enable the satellite to regain the preferred orientation within $\pm 0.003°$ in less than 25 sec when exposed to a micrometeorite impact[6] of $\simeq 5 \times 10^{-7}$ slug fps. The semipassive character of the system limits the peak power requirement to $\simeq 5$ w.

A comment concerning the Earth shadow, which would render the controller ineffective, is appropriate here. Mathematically, this would cause the generalized forces and hence the control moments to vanish periodically. Fortunately, for a geostationary orbit, the influence of shadow is confined to a quarter of the satellite's life-span, and even here, only during 5% of the orbital period. The results showed the controller performance to remain virtually unaffected.

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